Basic concepts in quantum physics

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Overview

- Introduction
- Superposition: beam splitters, randomness, single particle interference…
- The qubit and quantum parallelism
- Entanglement: two-particle interference, correlations…
- Non-locality: a game

Introduction
Historical overview

- 1900 Max Planck: black body radiation
- 1905 Einstein: photoelectric effect
- 1911 Niels Bohr: the hydrogen atom
- 1926 Heisenberg, Schrödinger…: definitive theory

"I think I can safely say that nobody understands quantum mechanics" Richard Feynman

Applications of quantum physics

- Atomic and nuclear physics
- Particle physics (eg. CERN)
- Condensed matter physics (Semi and superconductors…)
- Optics (Laser…)
- Chemistry
- Cosmology

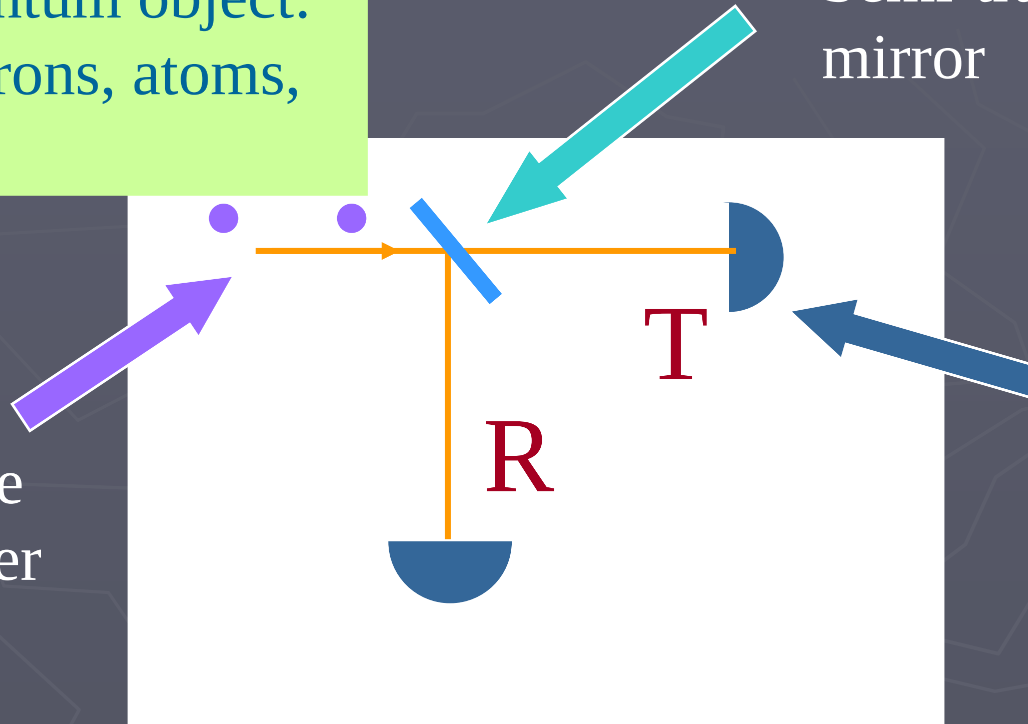
Superposition

Three experiences

Experience #1

Particle = quantum object: photons, electrons, atoms, molecules…

Semi-transparent mirror



Stream of particles, one after the other

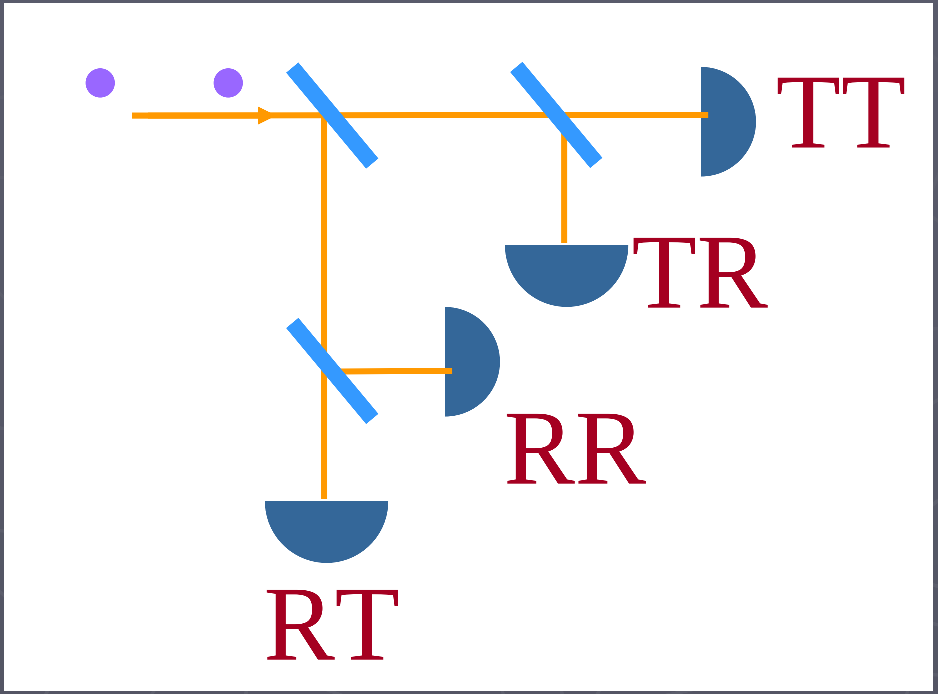
Detector (counter)

Two possible paths, reflected (R) and transmitted (T)
Each particle is indivisible: detected in either R or T
The outcome for each particle is random.
Probabilities: P(T) = P(R) = 1/2.

???

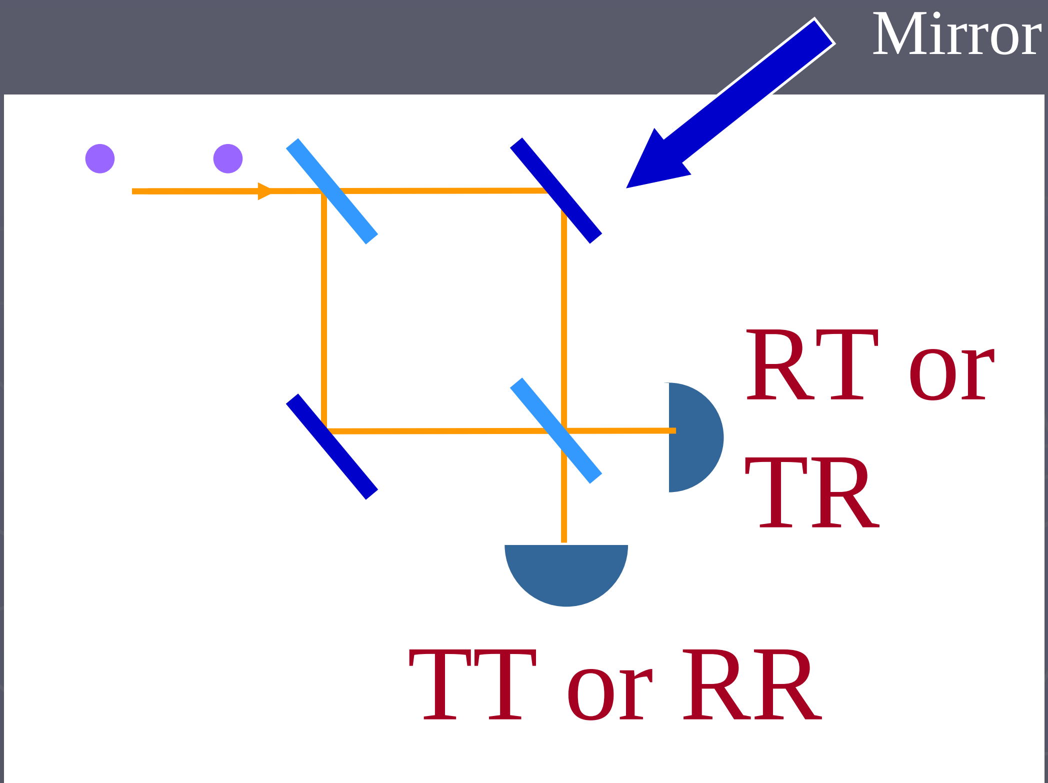
No matter how much we know

Experience #2



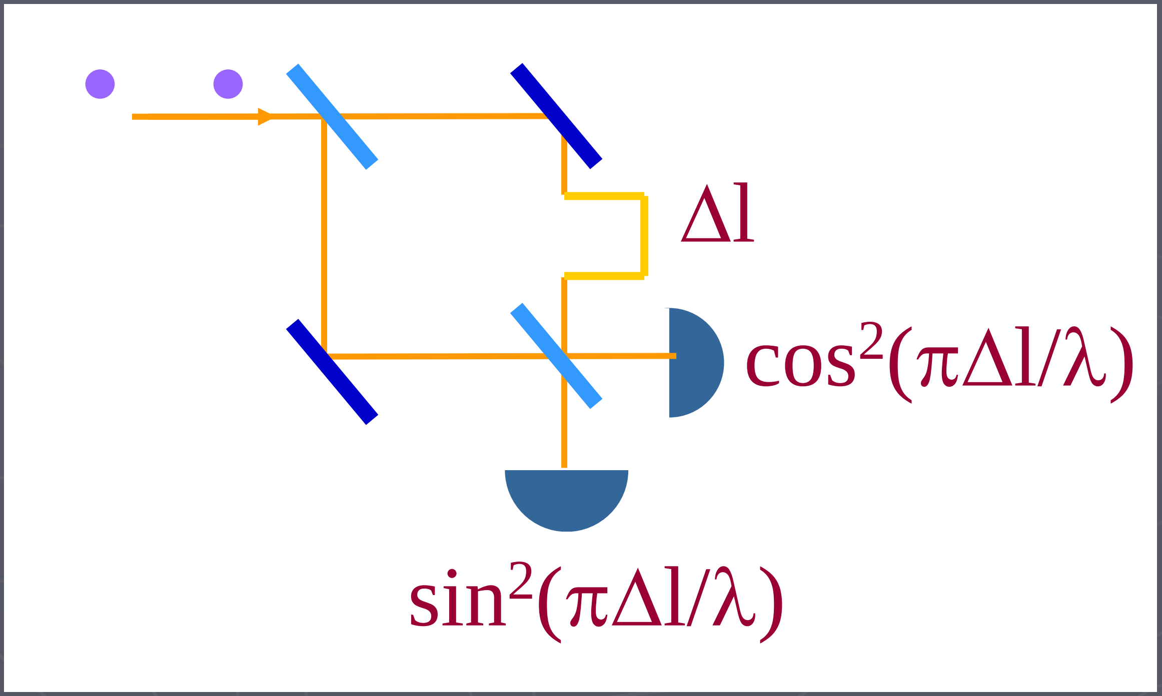
Four possible paths, transmitted twice (TT)...
Probabilities: P(TT) = P(TR) = P(RR) = P(RT) = 1/4.

Experience #3



Again four different paths, transmitted twice (TT)...
Probabilities: P(TT) = P(TR) = P(RR) = P(RT) = 1/4?
We observe **P(TT or RR) = 0, P(RT or TR) = 1**.
Here RT est indistinguishable from TR etc.

Experience #3 (modified)



Example: $\Delta l=\lambda/2$: **P(TT or RR) = 1, P(RT or TR) = 0**. ???

Changing a single path influences all the particles!

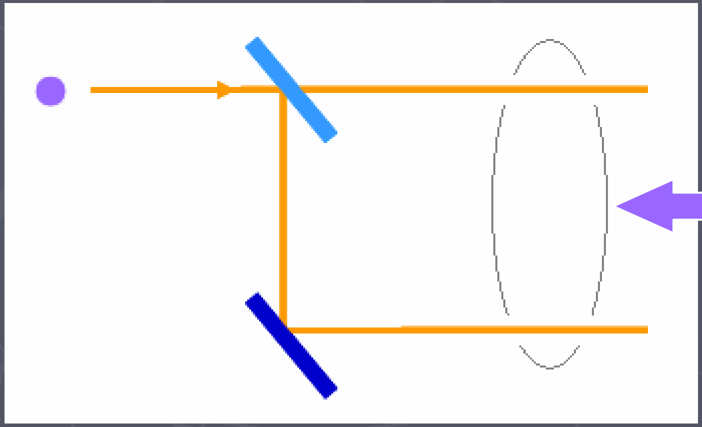
⇒ Every particle explores all possible paths

Single particle interference Observations

- Each particle explores all possible paths (delocalised), as a wave.
- Each particle is indivisible at the time of detection.
- If several different possibilities (paths) aren't distinguishable, then we observe interference effects.
- Single particle interference.

The quantum bit

The particle at times is in two paths simultaneously.
We then talk of a superposition, of the particle being in the reflected path and the transmitted path.



$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|R\rangle + |T\rangle\right)$$

State of the particle

General form

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

Probability amplitudes (complex numbers)

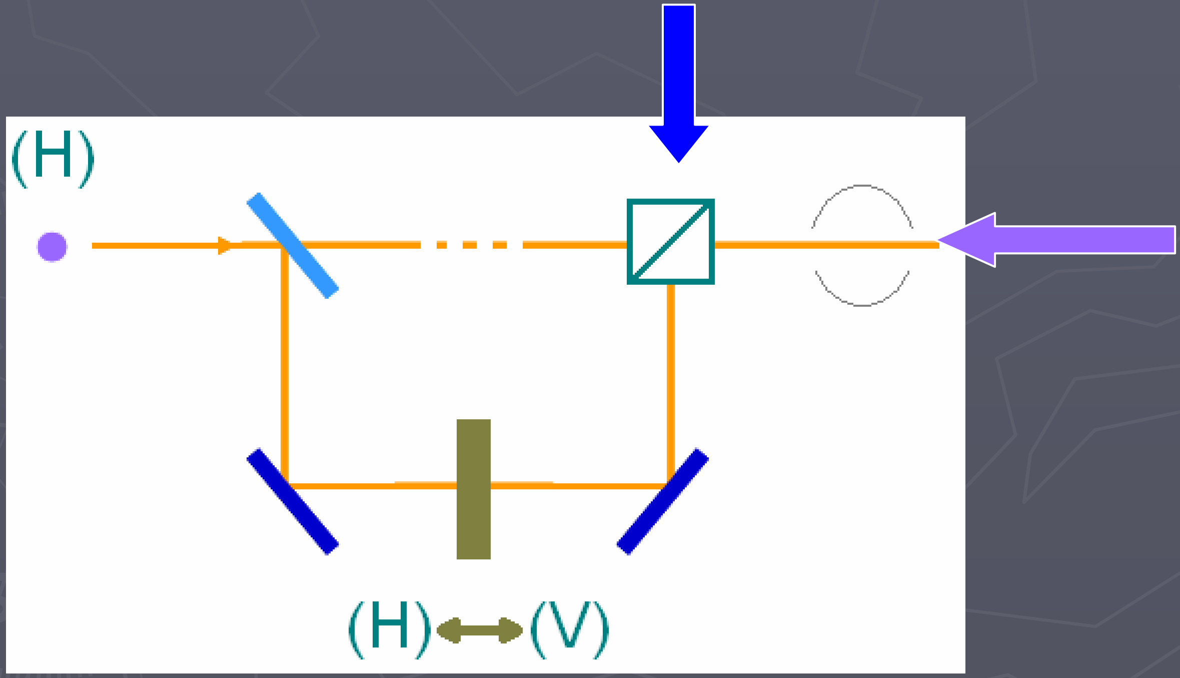
Probabilities: $|\alpha|^2, |\beta|^2$ Associated to the different measurement outcomes

Normalization: $|\alpha|^2 + |\beta|^2 = 1$

Different physical quantities

From path (position) encoding to polarization encoding.

Polarizing beam splitter



$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle + |V\rangle\right)$$

Multiple qubits

Suppose we decide to look at the quantum state of two qubits:

$$|\psi\rangle_1 = \alpha|0\rangle_1 + \beta|1\rangle_1$$

$$|\psi\rangle_2 = \gamma|0\rangle_2 + \delta|1\rangle_2$$

This state can be written as:

$$|\psi\rangle_{12} = \alpha\gamma|00\rangle + \alpha\delta|01\rangle + \beta\gamma|10\rangle + \beta\delta|11\rangle$$

So for n qubits... 2^n Possible states!!

Quantum gates

- One qubit gates, e.g. NOT-gate

$$\begin{aligned} |0\rangle &\rightarrow |1\rangle \\ |1\rangle &\rightarrow |0\rangle \end{aligned}$$

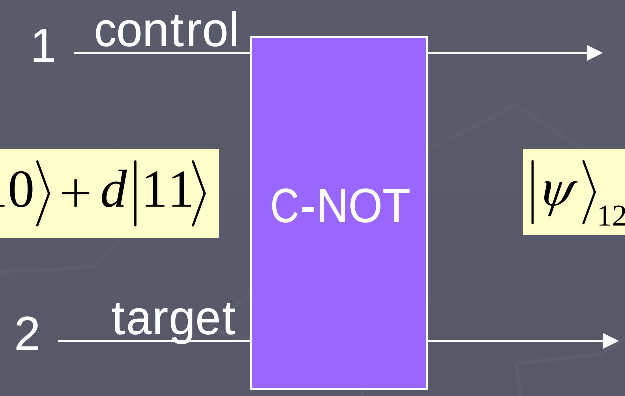
$$|0\rangle \xrightarrow{\boxed{\text{NOT}}} |1\rangle$$

- Typically quantum, e.g. Hadamard (H) gate

$$\begin{aligned} |0\rangle &\rightarrow \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) \\ |1\rangle &\rightarrow \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right) \end{aligned}$$

- two qubit gates, e.g. C-NOT gate

$$|00\rangle \to |00\rangle$$
$$|01\rangle \to |01\rangle$$
$$|10\rangle \to |11\rangle$$
$$|11\rangle \to |10\rangle$$



$$|\psi\rangle_{12} = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$$

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- Typically quantum e.g. C-Phase gate

$$|00\rangle \to |00\rangle$$
$$|01\rangle \to |01\rangle$$
$$|10\rangle \to |10\rangle$$
$$|11\rangle \to -|11\rangle$$

$$|\psi\rangle_{12} = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle \Rightarrow |\psi\rangle_{12} = a|00\rangle + b|01\rangle + c|10\rangle - d|11\rangle$$

Quantum parallelism

- Fundamental feature of many quantum algorithms.
- Roughly speaking, a computer is able to evaluate a function f(x) for many different values of x simultaneously.
- To illustrate this, suppose f(x) is a function mapping one bit to one bit.

$$f(x):\{0,1\}\to\{0,1\}$$

- Consider a two qubit quantum computer starting off in the state:

$$|x\rangle|y\rangle$$

data target

- We can transform the state as:

$$|x\rangle|y\rangle \rightarrow |x\rangle|y \oplus f(x)\rangle$$

- If the data register is initially prepared in the superposition state we saw earlier and the target register in the state $|0\rangle$:

$$\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right)|0\rangle \rightarrow \frac{|0\rangle|f(0)\rangle + |1\rangle|f(1)\rangle}{\sqrt{2}}$$

- The state contains information about BOTH f(0) and f(1)!!

Quantum parallelism

- However this parallelism is *not* immediately useful.
- In this example, a measurement of the qubits will give us only *either* f(0) or f(1)..
- A classical computer can do this easily.
- Quantum computation requires something more than just quantum parallelism, it requires the ability to *extract* information about more than one value of f(x) from *superposition* states.

- Considering the same function f(x), if we set the data and target registers as two different superpositions and operate the function we can map:

$$\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)\to|f(0)\oplus f(1)\rangle\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)$$

- So by measuring the first qubit, we may determine f(0)+f(1) in *only one* evaluation of f(x), a global property of that function.
- Would obviously require two evaluations on a classical computer.

Entanglement

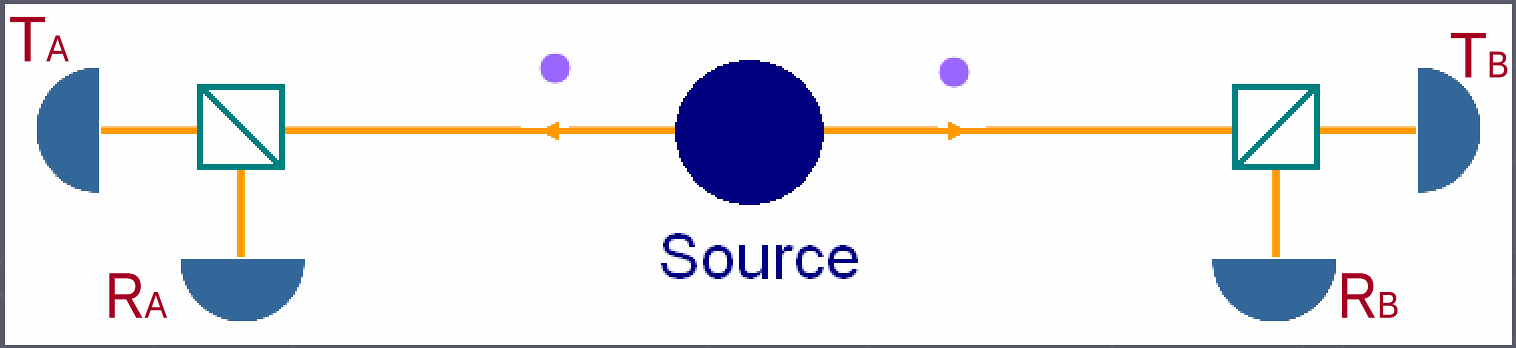
- The 'spooky action at a distance' as referred to by Einstein.
- Lets consider a source of entangled particles, for example photons with entangled polarizations.

$$\left|\psi\right\rangle = \frac{\left|H\right\rangle\left|H\right\rangle + \left|V\right\rangle\left|V\right\rangle}{\sqrt{2}}$$

- Notice how the overall state of the system is perfectly well defined, while the behavior of the individual particles is random.
- The overall state cannot be written as two independent systems.

Experience #4

Alice



Bob

$$|\psi\rangle_{AB} = \left(|H\rangle_A |H\rangle_B + |V\rangle_A |V\rangle_B\right)/\sqrt{2}$$

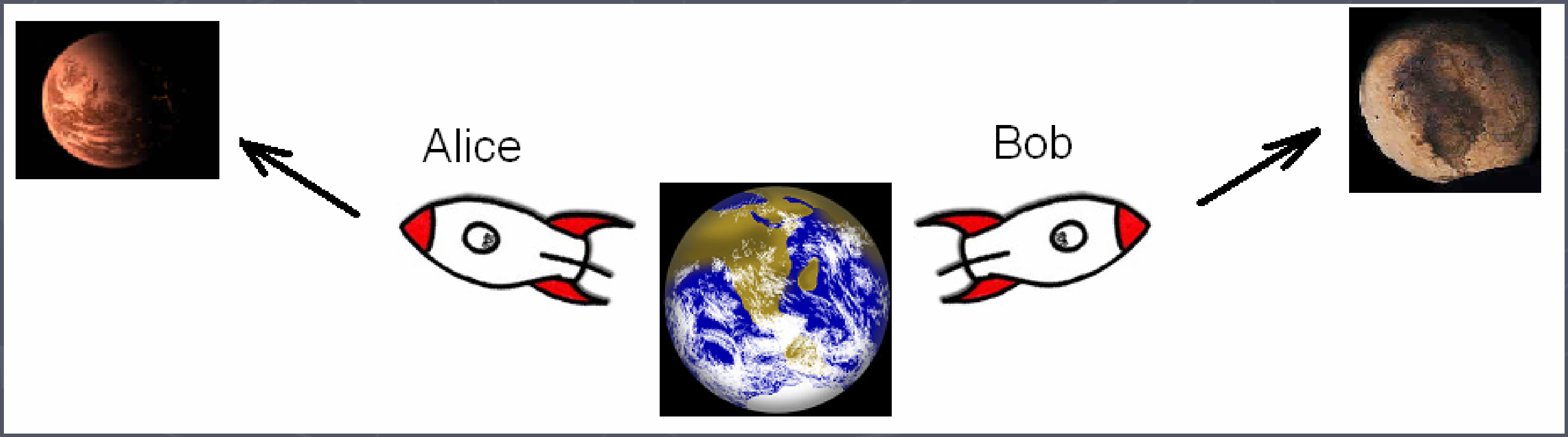
- Probabilities: $P(T_A) = P(R_A) = P(T_B) = P(R_B) = ½$
- Both Alice and Bob observe random results and *cannot* predict the measurement outcomes.
- However **$P(T_A R_B) = P(R_A T_B) = 0$,**
- And **$P(T_A T_B) = P(R_A R_B) = 1/2$**.

It's the 'same' randomness!!

But *non-signaling*

Non-locality: a game

- Pairs of participants (say Alice and Bob) are sent to different planets in different solar systems (say).
- Far enough not to be able to communicate during the time the game takes place.



Rules

- Alice's referee chooses (at random) one of two boards: right or left

Left	Right

- Alice writes a '+' or a '-', on that board

- Bob's referee chooses (at random) one of two boards: right or left

Left	Right

- Bob writes a '+' or a '-', on that board

Winning results

Alice	Bob
L	L
L	R
R	L
R	R

L L, L R, R L → **Coincidence** +|+ or -|-

R R → **Anti-coincidence** +|- or -|+

- Before leaving the Earth, they agree on a strategy. Their memory can be seen as a *classical* correlation.
- The optimal classical strategy Alice and Bob have enables them to win 3/4 of the time.
- One outputs a fixed sign in all cases the other a different sign for each board. e.g.

Alice: + for Left
- for Right

Bob: + for Left and Right

L	**L**	**+**	**+**	✓
L	**R**	**+**	**+**	✓
R	**L**	**-**	**+**	✗
R	**R**	**-**	**+**	✓

The quantum strategy

- Now Alice and Bob share an entangled state of the form $|\psi\rangle_{AB} = \left(|0\rangle_A |0\rangle_B + |1\rangle_A |1\rangle_B\right) / \sqrt{2}$
- They both agree on 2 measurements they can each perform, onto their particle. Each measurement corresponds to a choice of board (Right of Left). Remember the referee randomly chooses one.
- Each measurement has two outcomes, say either + or -. This is what they then write on the board.
- These measurement outcomes are *random*.

- Using this strategy, they will win the game with probability

$$P = \frac{2+\sqrt{2}}{4} \approx 0.85 > 0.75$$

Non-local effect

Classical/Local limit

- There are correlations even *stronger* than quantum correlations, which would still be non-signaling, and enable Alice and Bob to win the game all the time.

Conclusion

- Quantum systems can be in a (coherent) superposition of different states.
- These states can be used to encode information and lead to quantum parallelism.
- Interference can be used to extract useful classical information in fewer computation steps.
- Indeterminism and superposition lead to entanglement.
- Non-local features can be observed.